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$$\therefore PD=CD-CP=\frac{q+1+\sqrt{[p^2+(q-1)^2]}}{2}.\quad \therefore PM^2=CP.PD=-\tfrac{1}{4}p^2+q.$$

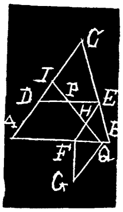
$\therefore (OP+PM\sqrt{-1})(OP-PM\sqrt{-1})=OP^2+PM^2=\frac{1}{4}p^2-\frac{1}{4}p^2+q=q$, and $(OP+PM\sqrt{-1})+(OP-PM\sqrt{-1})=2OP=-p$.

230. Proposed by **SAUL EPSTEEN**, Ph. D., The University of Chicago.

Cut off a given area S from a given triangle ABC by means of a line passing through a given point P, (i) when P is on a side of ABC, (ii) when P is within ABC ($S<$area ABC). [For the case P outside of ABC see problem 218].

I. Solution by **J. SCHEFFER**, Kee Mar College, Hagerstown, Md.

(i) The point P lies on the side AB. If S is smaller than $\triangle ACP$, find h from the proportion $AP:m=2m:h$, when $S=m^2$.



Lay off h perpendicularly to AB between the sides AC and AB. Connect the point where it strikes AC with P, and $\triangle APQ$ will be the required portion cut off. If $S>ACP$, take BP instead of AP.

(ii) Point P within the triangle.

Draw DE parallel to AB through P. Make parallelogram $LEFA=S$. On F erect perpendicular $FG=DP$, make $GQ=PE$, and connect Q with P extending to I, then will IQ be the required line. For

$$\frac{\triangle FQH}{\triangle PHE}=\frac{PE^2-DP^2}{PE^2}=1-\frac{DP^2}{PE^2}=1-\frac{\triangle DPI}{\triangle PHE}.$$

$\therefore \triangle FAH=\triangle PHE-\triangle DPI$, or $\triangle FQH+\triangle DPI=\triangle PHE$.

$\therefore \triangle FQH+\triangle DPI+AFHPD=\triangle PHE+AFHPD$.

$\therefore \triangle AQI=AFED=S$.

II. Solution by **G. B. M. ZERR**, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle altitude h, P the point on side AB, Q the point within. Lay off $AD=2S/h$ and draw BD; then $ABD=S$.

(i) Draw PD, and parallel to PD draw BM; join PM. The triangle PDM =triangle PDB.

$\therefore PDM+ADP=APM=ADP+PDB=ADB=S$.

(ii) Through Q draw HG parallel to AC. Draw HD, and parallel to HD draw BE. Join HE. Then $HED=HBD$, and $AEH=ADB$. Bisect AE at K, and draw KF parallel to AH. Then $AKFH=AEH=ADB$. At K draw KL perpendicular to AC, and make $KL=HQ$, $LM=QF$.

$\therefore KL^2+KM^2=LM^2$. $\therefore HPQ+KNM=QFN$.

$\therefore PAM$ is the required line. For $PAM=AKFH=ADB=S$.

III. Solution by **G. W. GREENWOOD**, M. A. (Oxon), Lebanon, Ill.

The following universal construction is Euclidean, but the proof is based on conics.

From the angle A cut *any* triangle XAY equal to the given area. Construct the triangle ZAZ' where Z and Z' lie on the sides of the angle A, and such that $AZ=AZ'=\sqrt{(AX.AY)}$. Then the area of this triangle is also equal to the given area. All lines cutting from the angle A a triangle having this area are tangent to an hyperbola having A as center and the sides of the angle as asymptotes. ZZ' is the tangent at a vertex. The circle with A as center and tangent to ZZ' is the auxiliary circle. It will cut the sides of the angle A in points which we will call M, N. At these points erect perpendiculars to the sides of the angle; these will intersect in the corresponding focus of the hyperbola S, say. Construct the circle with PS as diameter, and if this intersects the auxiliary circle, call one point of intersection Q. The angle $\angle SQP$ is right, and hence PQ is a tangent to the hyperbola, and so cuts off from the angle A the required area. There is no solution when there is no point Q, nor, according to the limitations of the question, when the segment PQ intersects the segment BC.

We can treat the angles B and C in like manner.

Also solved by Elmer Schuyler, and A. H. Holmes.

231. Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.

A man starts from the vertex, A, of a right isosceles triangle ABC, right-angled at A, and walks to D, the middle point of BC; from D to E, the middle point of AC; from E to F, the middle point of AD; from F to G, the middle point of DE; from G to H, the middle point of EF; from H to I, the middle point of FG; from I to J, the middle point of HI; and so on *ad infinitum*. Find the coördinates of his limiting position. [Suggested by Dr. Crawley].

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

From J he walks to K, the mid-point of HI. He then performs the same journeys in the triangle KIJ as in ABC, and so on. The sides of KIJ are $\frac{1}{16}$ the length of the sides of ABC. Taking A as origin, the coördinates of K are $\frac{9}{16}AD$, and $\frac{3}{16}DC=\frac{3}{16}AD$. Hence the coördinates of his limiting position are

$$\tfrac{9}{16}AD\left(1+\frac{1}{16}+\frac{1}{16^2}+\frac{1}{16^3}+\frac{1}{16^4}+\dots\dots\right)=\tfrac{9}{15}AD=\tfrac{3}{5}AD,$$

$$\tfrac{3}{16}AD\left(1+\frac{1}{16}+\frac{1}{16^2}+\frac{1}{16^3}+\frac{1}{16^4}+\dots\dots\right)=\tfrac{3}{15}AD=\tfrac{1}{5}AD.^*$$

II. Solution by A. H. HOLMES, Brunswick, Maine.

Draw the lines AD, DE, EF, FG, GH, HI, IJ, JK, and KL, according to the directions of the problem. Join AG and this line will pass through the point K. This is easily seen by drawing GN parallel to DC, meeting AD in N, and KM parallel to DC, meeting AD in M, and considering the similar triangles

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*Taking D as the origin, DC and DA as the axes, the coordinates of the limiting point become $\frac{1}{5}AD$, $\frac{2}{5}AD$. This result agrees thus with the one in Solution II by Mr. Holmes. Ed.